Appendix

Grid-based estimators of second-order statistics

The study area is divided into a grid of cells. The size of the cells should be defined as the minimal resolution necessary to respond to the scientific question to be answered and is limited by the measurement uncertainty of point coordinates (Wiegand & Moloney 2004). The calculation of point-to-point distances necessary for estimation of second-order statistics is then based on distances between cells, and counting cells and points in cells. Wiegand & Moloney (2004) proposed a simple grid-based estimator of the bivariate *K*-function for arbitrarily shaped study regions, which is based on the mean number of type 2 points found in (complete or incomplete) circles of radius *r* around all type 1 points *k* [= $P_{12}(r)$], divided by the area of these circles [= A(*r*)]:

$$\lambda_2 \hat{K}_{12}(r) = \pi r^2 \frac{P_{12}(r)}{A(r)} = \pi r^2 \frac{\frac{1}{n}\sum_{k=1}^{n_1} \mathbf{Points}_2[C_{1,k}(r)]}{\frac{1}{n_1}\sum_{k=1}^{n_1} \mathbf{Area}[C_{1,k}(r)]} \qquad \text{(A1)}$$

The circles are incomplete if the focal point has a distance smaller than *r* to the border of the study region, λ_2 is the intensity of pattern 2, n_1 is the number of type 1 points in the study region, $C_{1,k}(r)$ is the circle with radius *r* centred on the *k*th type 1 point, the operator. $\mathbf{Points_2}$[X] counts the points of the pattern 2 in a region X, and the operator **Area**[X] determines the total number of cells of the region X. Note that this estimator does not scale the number of points in incomplete circles to the expected number within complete circles as is commonly done in point pattern analysis. The grid-based estimator is therefore not affected by the problem that the weights may become unbounded if *r* becomes larger. The grid-based estimator of the *L*-function using equation A1 yields:

$$\hat{L}_{12}(r) = r\left(\sqrt{\frac{\lambda_2 \hat{K}_{12}}{\pi r^2}} - 1\right) = r\left(\sqrt{\frac{A}{n_2}\frac{P_{12}(r)}{A_1(r)}} - 1\right). \qquad \text{(A2)}$$

An analogous grid-based estimator of the bivariate pair-correlation function $g_{12}(r)$ is given by

$$\lambda_2 \hat{g}_{12}(r) = \frac{\frac{1}{n}\sum_{k=1}^{n_1} \mathbf{Points}_2[R^w_{1,k}(r)]}{\frac{1}{n_1}\sum_{k=1}^{n_1} \mathbf{Area}[R^w_{1,k}(r)]} \qquad \text{(A3)}$$

where $R^w{}_{1,k}(r)$ is the ring with radius *r* and width *w* centred on the *k*th type 1 point (Wiegand & Moloney 2004). For the univariate case, this estimator was used in Condit *et al.* (2000). The estimator equation (A3) involves a technical decision on the width *w* of the rings. Clearly, the use of rings that are too narrow will produce jagged plots, as not enough points will fall into the different distance classes. This problem does not occur with the *K*-function. On the other hand, if the rings are too wide, the pair-correlation function will lose the advantage that it can isolate specific distance classes. More sophisticated estimators of the pair-correlation function use kernel functions such as the Epanečnikow kernel with a bandwidth *h* instead of rings with width *w*. For a detailed discussion of estimators of the pair-correlation function see Stoyan & Stoyan (1994), Stoyan & Stoyan (1996), and references therein.